PACE

TR-10

ANALOG COMPUTER

APPLICATION NOTES

NUMBER 1

DESIGN OF AN AUTOMOBILE SUSPENSION SYSTEM

Problem Description:

To investigate the response of an automobile suspension system for selected disturbances. The system response to these disturbances for various values of the system design parameters are obtained. The most suitable values of the system parameters are determined by selecting the desirable response from the computer solution.

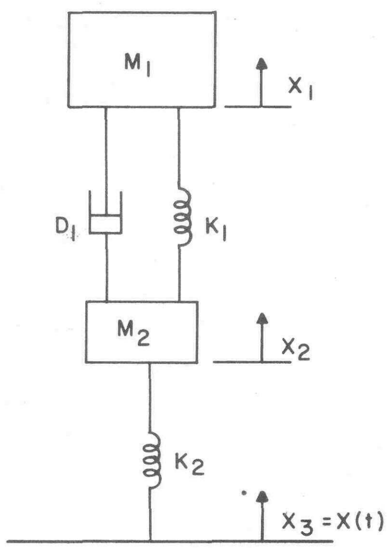


Figure 1: Simplified representation of a single wheel of an automobile suspension system.

Where:

M_1 = One quarter of mass of automobile
M_2 = Mass of the wheel and axle
K_1 = Spring constant of main auto spring
K_2 = Spring constant of tire (assumed linear)
D_1 = Shock absorber damping constant
x_1 = Displacement of auto body
x_2 = Displacement of wheel
x_3 = Roadway profile displacement

Physical Constants:

M_1 = 25 slugs
M_2 = 2 slugs
K_1 = 1000 lbs/ft.
K_2 = 4500 lbs/ft.
D_1 = Variable

Initial conditions and forcing functions:

$$x_1 = x_2 = \frac{dx_1}{dt} = \frac{dx_2}{dt} = 0 \text{ at } t = 0$$

$$x_3 = x(t)$$

System Equations:

The differential equations of motion of the system are derived by equating the forces acting upon the mass involved in the system.

They are:

$$\frac{d^2x_1}{dt^2} = -\frac{D_1}{m_1}\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) - \frac{K_1}{m_1}(x_1 - x_2)$$

$$\frac{d^2x_2}{dt^2} = -\frac{D_1}{m_2}\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) - \frac{K_1}{m_2}(x_2 - x_1) - \frac{K_2}{m_2}(x_2 - x_3)$$

Scaled Equations:

Assume the following maximum values of the scaled variables to assure that the computer problem voltages are as high as possible but less than ± 10 volts.

$[5 \frac{d^2x_1}{dt^2}]$	$[0.8 \frac{d^2x_2}{dt^2}]$
$[5 \frac{dx_1}{dt}]$	$[0.8 \frac{dx_2}{dt}]$
$[50 x_1]$	$[50 x_2]$

The scaled equations then become:

$$[5\ddot{x}_1] = -\left(\frac{D_1}{25}\right)[5\dot{x}_1] + \left(\frac{D_1}{4}\right)[0.8\dot{x}_2] - (4.0)[50x_1] + (4.0)[50x_2]$$

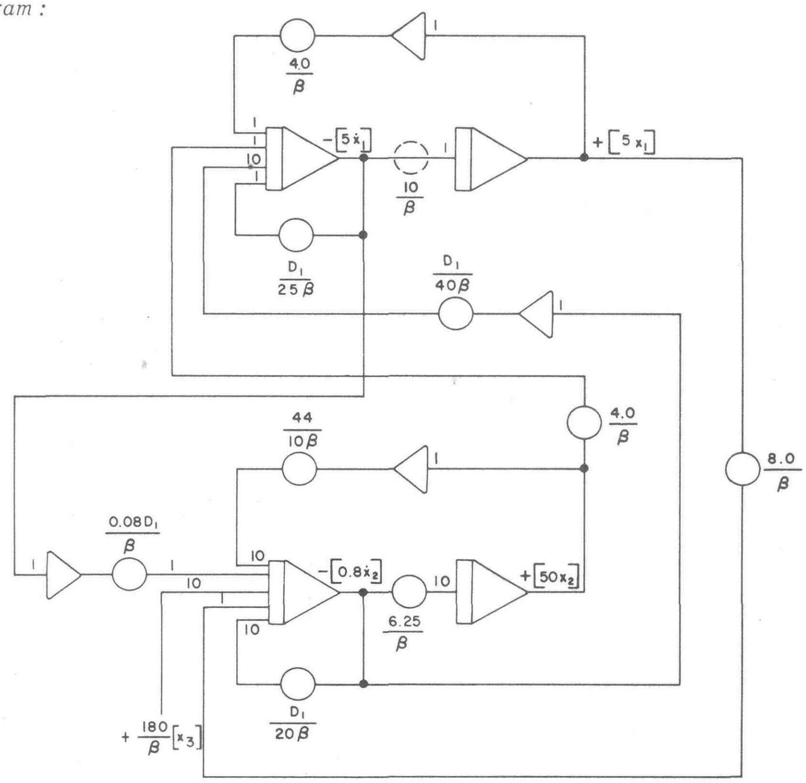
$$[0.8\ddot{x}_2] = -\left(\frac{D_1}{2}\right)[0.8\dot{x}_2] + (0.08 D_1)[5\dot{x}_1] - (44)[50x_2] + (8)[50x_1] + (1800)[x_3]$$

Where $\ddot{x} = \frac{d^2x}{dt^2}$; $\dot{x} = \frac{dx}{dt}$

( ) = Combination potentiometer settings and amplifier gains.

[ ] = Voltage outputs from computing components.

Computer Diagram:



Each input to every integrator is multiplied by a factor $\frac{1}{\beta}$ to change the speed with which the system behavior is obtained on the computer. In this example $\beta = 10$ appears appropriate and the computer circuit behavior will be ten times slower than that of the suspension system.

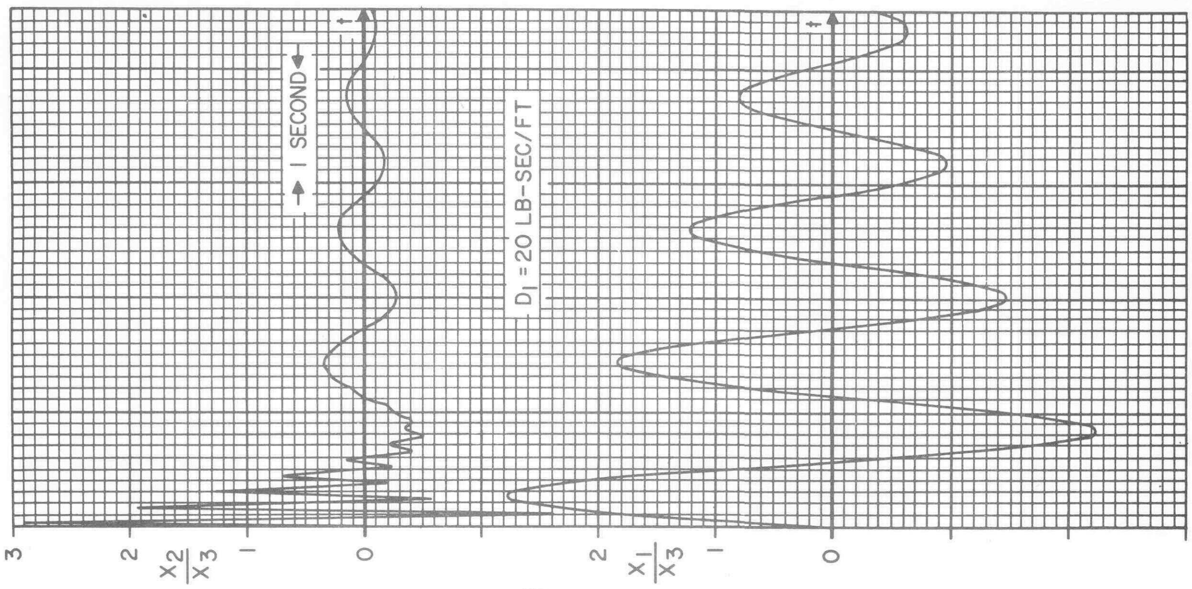


Figure 1a.

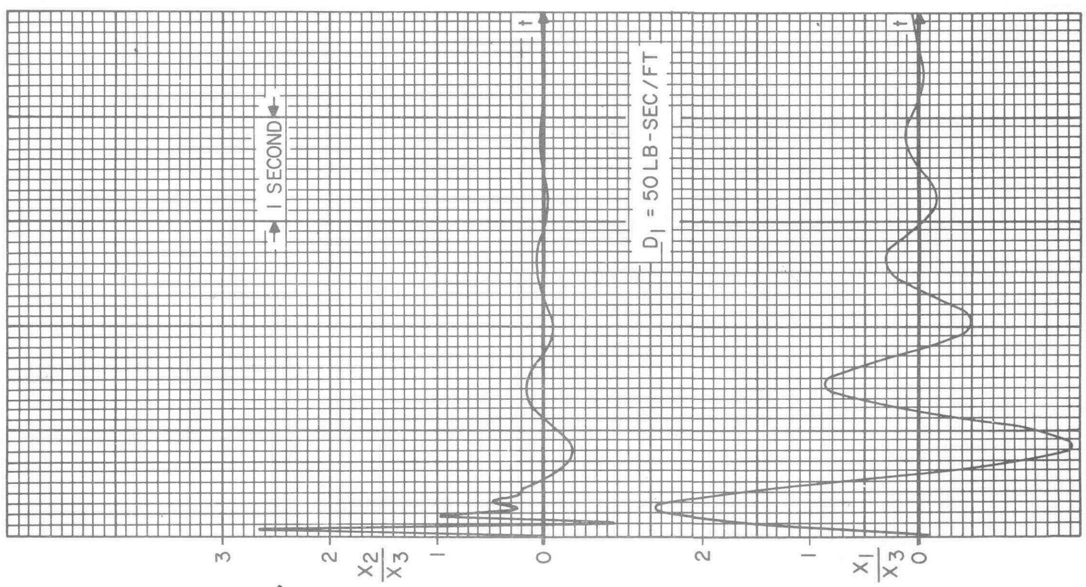


Figure 1b.

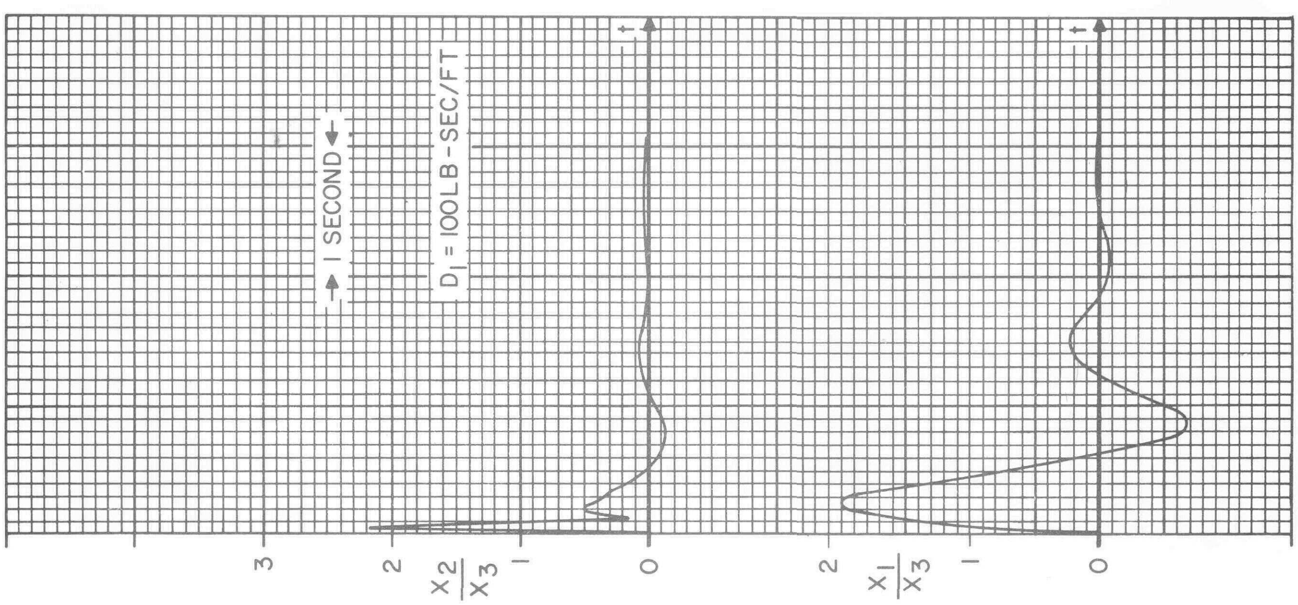


Figure 1c.

Displacement of auto body $(\frac{X_1}{X_3})$ *and wheel* $(\frac{X_2}{X_3})$ *for a deflection in road profile* (X_3 *is a square pulse of 35 milliseconds duration.)*

PACE TR-10 ANALOG COMPUTER

APPLICATION NOTES

NUMBER **2**

CALCULATION OF HEAT TRANSFER BY NATURAL CONVECTION[1]

Problem Description:

To solve a natural convection heat transfer problem, the solution will consist of the temperature and velocity distribution for various Prandtl numbers. The quantity of heat which is transferred will also be calculated.

Equations:

The Navier-Stokes equation and the Fourier equation can be written for natural convection heat transfer from a vertical flat plate in the following manner[2].

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial^2 \psi}{\partial y^2} = v \frac{\partial^3 \psi}{\partial y^3} + g \beta \theta_s \phi$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} = \alpha \frac{\partial^2 \phi}{\partial y^2}$$

with boundary conditions

$$\frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0 \quad \phi = 1 \quad \text{At } y = 0$$

$$\frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0 \quad \phi = 0 \quad \text{At } y = \infty$$

Using substitutions suggested by Pohlhausen[1] of:

$$\epsilon = C \frac{y}{x^{1/4}}$$

where

$$C = \left(\frac{g \beta \theta_s}{4 v^2} \right)^{1/4}$$

and $\eta(\epsilon) = \phi(x, y)$

$$Z(\epsilon) = \frac{\psi(x, y)}{4 v c\, x^{3/4}}$$

The original equations reduce to the following system of ordinary differential equations:

$$\dddot{Z} + 3 Z \ddot{Z} - 2 \dot{Z}^2 + \eta = 0$$

$$\ddot{\eta} + 3 P_R Z \dot{\eta} = 0$$

With boundary conditions

$$Z = 0 \quad \dot{Z} = 0 \quad \eta = 1 \quad \text{At } \epsilon = 0$$

$$\dot{Z} = 0 \quad \ddot{Z} = 0 \quad \eta = 0 \quad \text{At } \epsilon = \infty$$

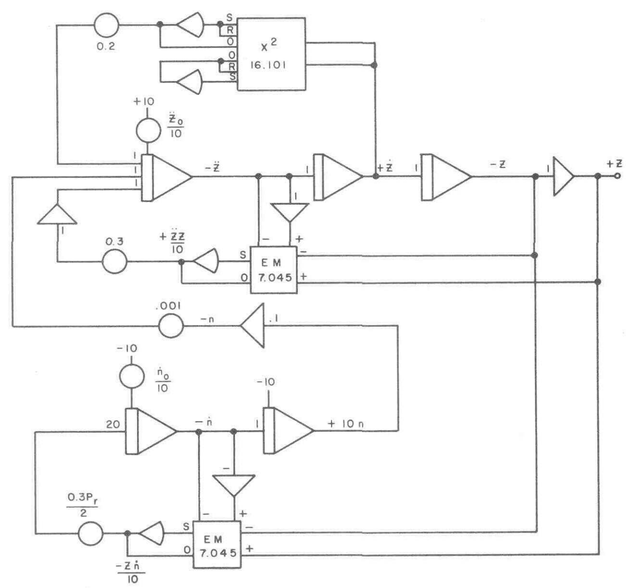
where the dots denote differentiation with respect to ϵ.

In order to solve these equations a substitution is made for the independent variable.

$$\epsilon = t/a \qquad a = 10$$

$$\frac{d}{d\epsilon} = a \frac{d}{dt} \text{ and } \frac{d^2}{d\epsilon^2} = a^2 \frac{d^2}{dt^2}$$

Computer Diagram:



Results of Analog Computer Solution:

The solution of the problem is one of trial and error in that all the initial conditions (the values of the functions at $\epsilon = 0$) must be known. Values for $\ddot{Z}$ and $\dot{\eta}$ at $\epsilon = 0$ are assumed and if the boundary conditions are satisfied, the assumed values are correct.

Figures 1 and 2 show the temperature and velocity distributions for various values of the Prandtl number as computed.

To calculate the quantity of heat which is transferred from the plate only the value of $\dot{\eta}$ at $\epsilon = 0$ is required. The desired values of the design parameters of interest can then be calculated using well known steady state heat transfer relationships!

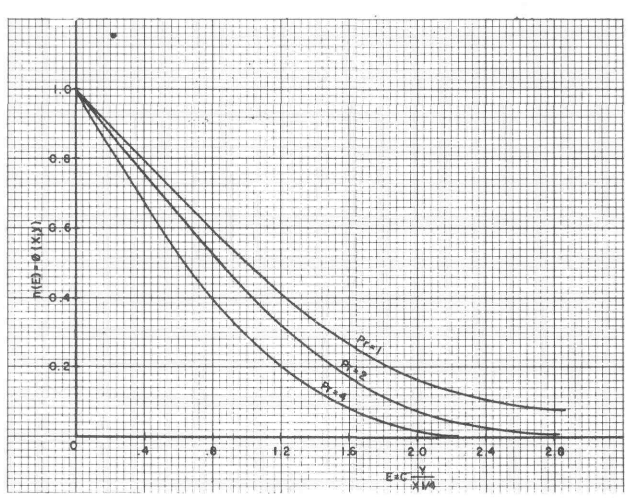


Figure 1: Heat transfer temperature profile.

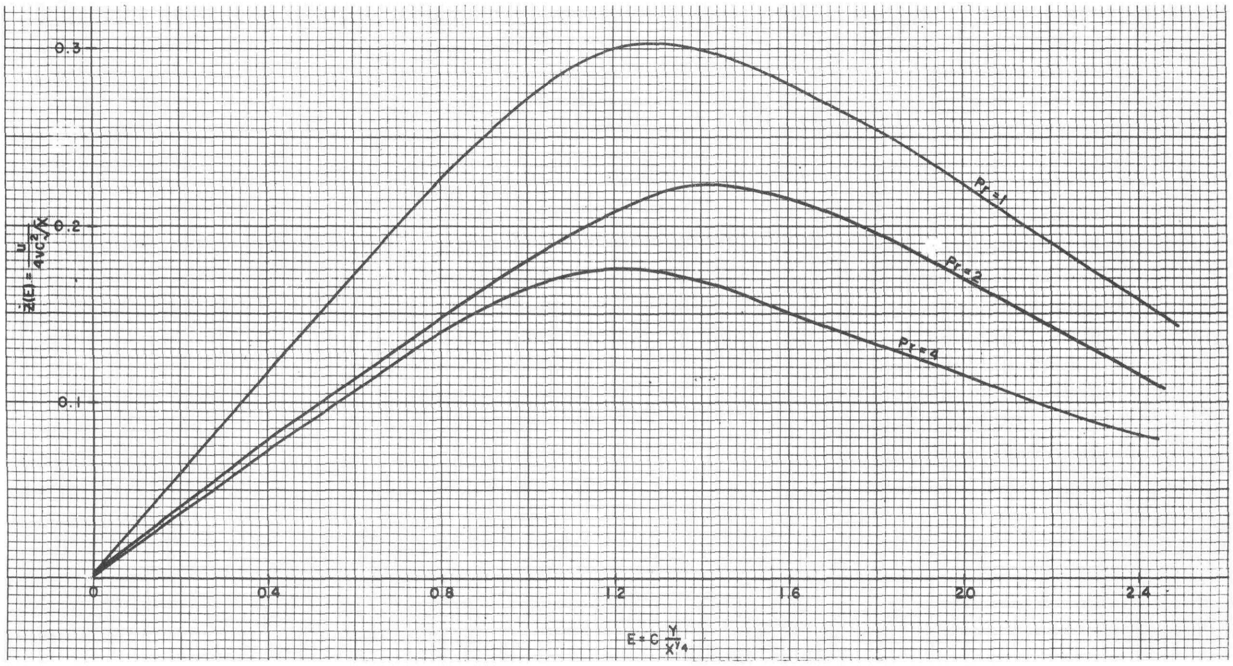


Figure 2: Heat transfer velocity profile.

Nomenclature:

$C = \left(\frac{g\,\beta\,\theta_s}{4\,v^2}\right)^{1/4}$

C_P = Heat capacity of fluid

$G_R = \frac{g\,\beta\,\theta_s L^3}{v^2}$ (Grassoff number)

g = Acceleration due to gravity

h = Over-all heat transfer coefficient.

k = Thermal conductivity of fluid

L = Height of plate

$Nu = \frac{hL}{k}$ (Nusselt number)

$P_R = \frac{C_P \mu}{k}$ (Prandtl number)

T = Temperature of fluid at point

T_∞ = Temperature of bulk fluid

T_P = Temperature of plate

$T_a = \frac{T_P + T_\infty}{2}$

u = Velocity in vertical direction

v = Velocity in horizontal direction

x,y = Vertical and horizontal coordinates of a point in fluid

$\alpha = \frac{k}{C_P \rho}$

$\beta = \frac{\rho_\infty - \rho_a}{\rho_a (T_a - T_\infty)}$

μ = Dynamic viscosity

v = Kinematic viscosity

ρ = Density of fluid

$\phi = \frac{T - T_\infty}{T_P - T_\infty}$

REFERENCES

1. This application is based upon an article by R.S.Schechter *(Analog Computer Calculates Heat Transfer)*, Petroleum Refiner, Vol. 36, No. 2 pp. 112 – 114, Feb. 1957.

2. Jakob, M.; "*Heat Transfer*", Vol. 1, Wiley and Sons, New York, 1950.

PACE
TR-10
ANALOG COMPUTER

APPLICATION NOTES

NUMBER 3

CALCULATION OF RADIAL VELOCITY OF ROTARY SPRAY DRIER

Problem Description:

To compute a curve of radial velocity of a particle vs radial distance from the center of a disc used in a spray drier. The results of the computation are then used to select the design parameters of a drier for a particular duty.

Equations:

The differential equation which describes the velocity of the particle is [1]:

$$V \frac{dV}{dr} + AV^3 - Br = 0$$

where

V = Radial Velocity

r = Radial distance from center of disc

A, B = Constants which in the case of laminar flow depend upon the angular velocity of disc, volumetric feed rate, vane height, and the density and viscosity of the liquid. For turbulent flow the same equation hold, with different value for constant A.

Physical Constants:

Mean values of

$$A = 1.66 \times 10^{-4}$$
$$B = 3.6 \times 10^{7}$$
$$r = 0 \text{ to } 5 \text{ inches}$$
$$V = 0 \text{ to } 6000 \text{ ft/min}$$

Computer Equations:

In order to solve this equation a substitution must be made for the independent variable r:

$$\text{let } r = t/a$$
$$\text{then } \frac{d}{dr} = a\frac{d}{dt}$$

The equation becomes with physical constants:

$$\frac{dV}{dt} = \frac{3.6 \times 10^7}{5V} r - \frac{1.66 \times 10^{-4} V^2}{5}$$

where $a = 5$

[1] See Perry *"Chemical Engineers Handbook"*, McGraw-Hill Book Co.

EAI
ELECTRONIC ASSOCIATES INC. *Long Branch, New Jersey*
PRINTED IN U.S.A. AF941
BULLETIN #AN 923

The computer will compute $10^{-3}V$, thus the equation becomes

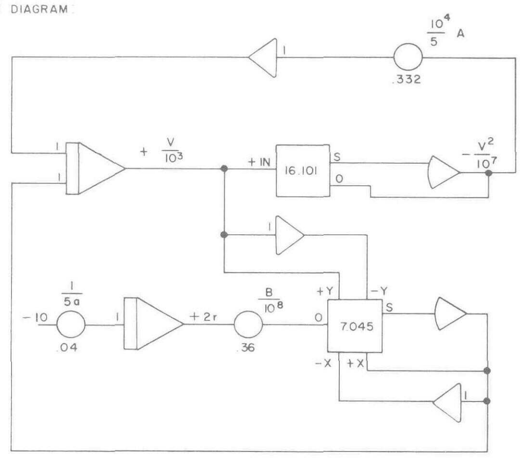
$$10^{-3}\frac{dV}{dt} = \frac{10^{-3}Br}{5V} - \frac{10^{-3}AV^2}{5}$$

Calculation of radius:

$$t = ra \qquad dr = \frac{1}{a}dt \qquad 2r = \frac{2}{a}\int dt$$

$$dt = a\,dr \qquad r = \frac{1}{a}\int dt$$

COMPUTER DIAGRAM



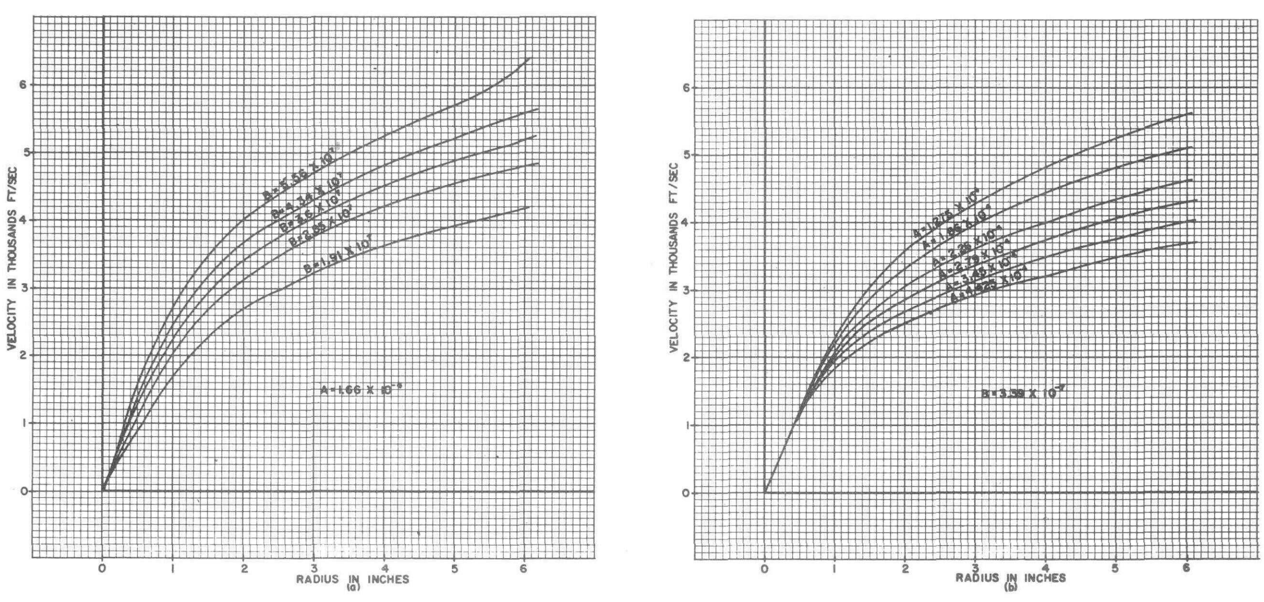


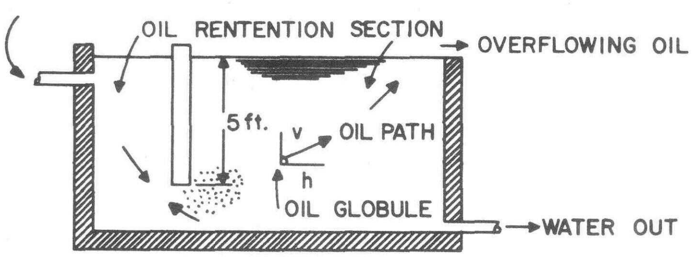
Figure 1: Plot of Radial Velocity as a Function of Drier Radius.

PACE
TR-10
APPLICATION NOTES
4
NUMBER
ANALOG COMPUTER

ANALYSIS OF THE FLOW PATH OF AN OIL GLOBULE

Problem Description:

It is required to separate an oil-water mixture by using a standard oil separator. The over-flowing level of oil is 5 feet above the lower end of the oil retention wall. Calculate the minimum length of separation section of the separator.



SCHEMATIC OF STANDARD OIL-WATER SEPARATOR

Equations:

To analyze the flow path of an oil globule, its velocity is resolved into vertical component v and horizontal component h. The horizontal velocity of an oil globule is assumed constant (throughout the separation section) and equal to the total volume flow rate divided by the cross-sectional area of the separator. The vertical velocity is variable and dependent upon a force balance which is derived as follows:

Inertial force of the oil globule

$$F_a = \frac{\pi D^3}{6} \rho_0 \frac{dv}{dt}$$

Buoyancy of the globule

$$F_b = \frac{\pi D^3}{6} (\rho_w - \rho_0) g$$

Drag force against the upflowing globule

$$F_d = \frac{f \left(\frac{\pi D^2}{4} \right) \rho_w v^2}{2}$$

Where

D = Average diameter of oil globule

v = Vertical velocity of oil globule

ρ_0 = Density of oil

ρ_w = Density of water

g = Acceleration due to gravity

f = Coefficient of viscous friction

The force balance becomes:

$$F_a = F_b - F_d$$

$$\frac{\pi D^3}{6} \rho_o \frac{dv}{dt} = \frac{\pi D^3}{6} (\rho_w - \rho_o) g - \frac{f \pi D^2 \rho_w v^2}{8}$$

$$\frac{dv}{dt} = \frac{\rho_w - \rho_o}{\rho_o} g - \frac{3 f \rho_w}{4D \rho_o} v^2$$

The vertical velocity v may by integrated with respect to time to give the vertical traverse distance of an oil globule within a given time.

$$s = \int_0^t v \, dt$$

The computer diagram is mechanized to solve for v and s. Since the total vertical traverse distance is known and the function of vertical velocity will be obtained from the computer, the above equation is employed to determine the time required for an oil globule to rise to the over-flowing surface. The minimum longitudinal length of the separation section equals the horizontal velocity h times the minimum retention time.

The variables are scaled as indicated below so that the computer problem voltages are as high as possible but less than ± 10 volts.

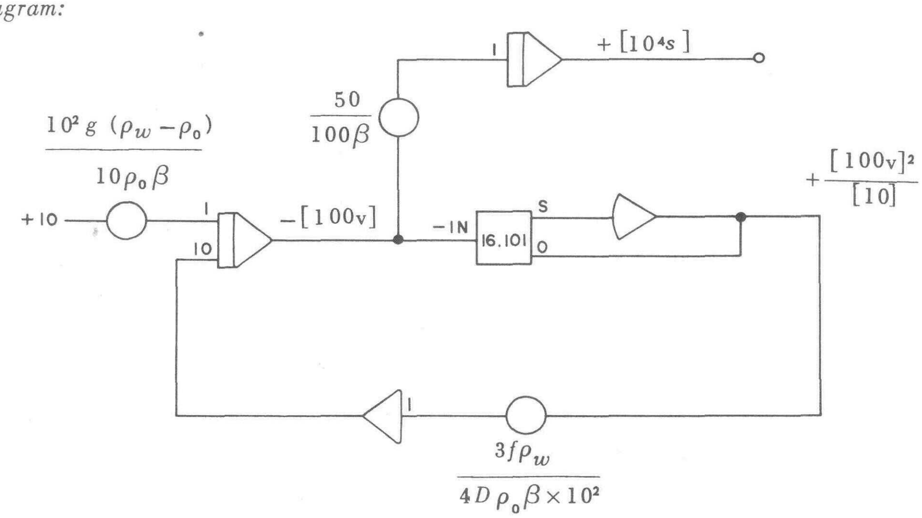
$[100 \frac{dv}{dt}]$ $\qquad$ $[50 \frac{ds}{dt}]$

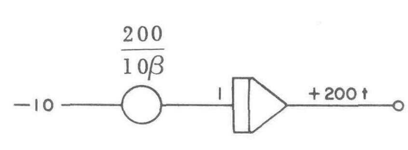
$[100v]$ $\qquad$ $[10^4 s]$

Scaled Equations:

$$[100v] = 10^2 g \frac{\rho_w - \rho_o}{\rho_o} - \frac{3 f \rho_w}{40D \rho_o} \frac{[100V]^2}{[10]}$$

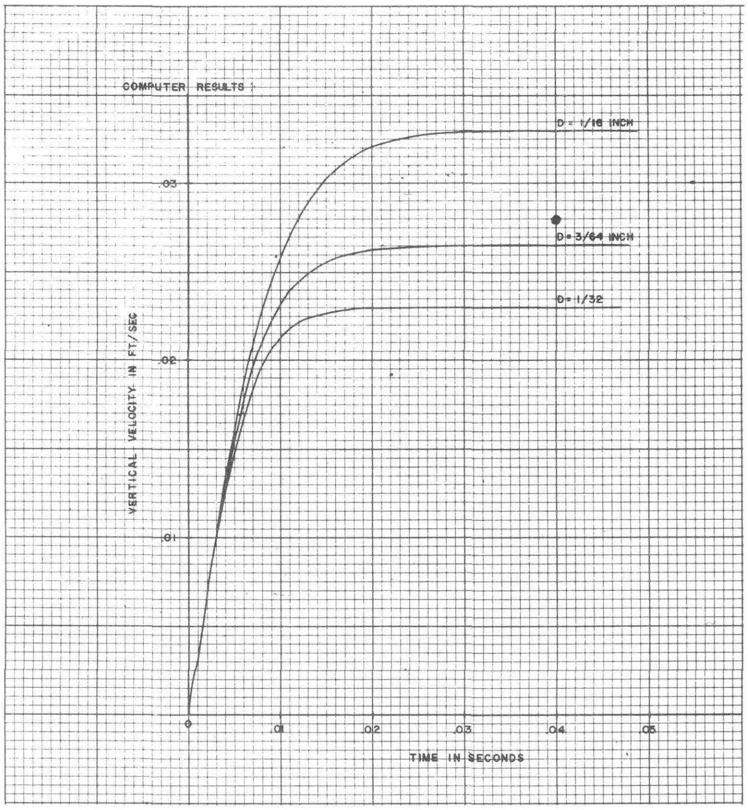
Computer Diagram:





Each input to every integrator is multiplied by a time scale factor $\frac{1}{\beta}$. In this example $\beta = 100$ and the computer solution will be 100 times slower than the behavior of the physical system.

Computer Results :



Plot of vertical velocity of oil globule with time.

PACE
TR-10
ANALOG COMPUTER

APPLICATION NOTES

NUMBER 5

INVESTIGATION OF A SIMPLE CHEMICAL REACTION

Problem Description:

The reaction of chemical component A to form component B, which in turn reacts to form component C, can be represented as follows:

$$A \xrightarrow{r_1} B \xrightarrow{r_2} C$$

where r_1 and r_2 are specific reaction rates. The reaction is assumed to take place at a constant temperature. Find the changes in concentration of A, B and C as a reaction time, (for the case of r_1 equal to unity and r_2/r_1 less than 1.0.)

Equations:

The reaction rate for such reactions may be expressed by the following differential equations, assuming no reverse reactions:

$$-\frac{dn_A}{dt} = r_1 n_A \qquad \text{rate of disappearance of A}$$

$$\frac{dn_B}{dt} = r_1 n_A - r_2 n_B \qquad \text{net rate of appearance of B}$$

$$\frac{dn_C}{dt} = r_2 n_B \qquad \text{rate of formation of C}$$

Where n_A, n_B and n_C are mole fractions. The mole fraction of a component is defined as the number of moles of that component divided by the sum of the number of moles of all components.

Therefore $0 < n_A,\ n_B,\ n_C < 1.0$

$$n_A = 1.0 \text{ at } t = 0$$

Scaled Equations:

The variables are scaled as shown below to assure that the voltages representing their maximum values are as large as possible within the limits of ± 10 volts.

$$[10\frac{dn_A}{dt}] \qquad [10\frac{dn_B}{dt}] \qquad [10\frac{dn_C}{dt}]$$

$$[10n_A] \qquad [10n_B] \qquad [10n_C]$$

The scaled equations then become

$$-[10\frac{dn_A}{dt}] = r_1\ [10n_A]$$

EAI ELECTRONIC ASSOCIATES INC. Long Branch, New Jersey

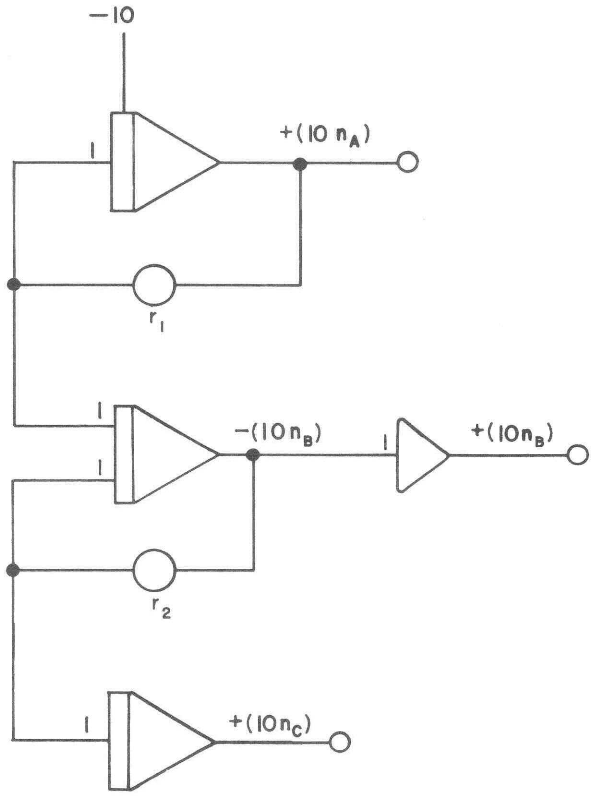
PRINTED IN U.S.A. AF941

BULLETIN # AN 925-1

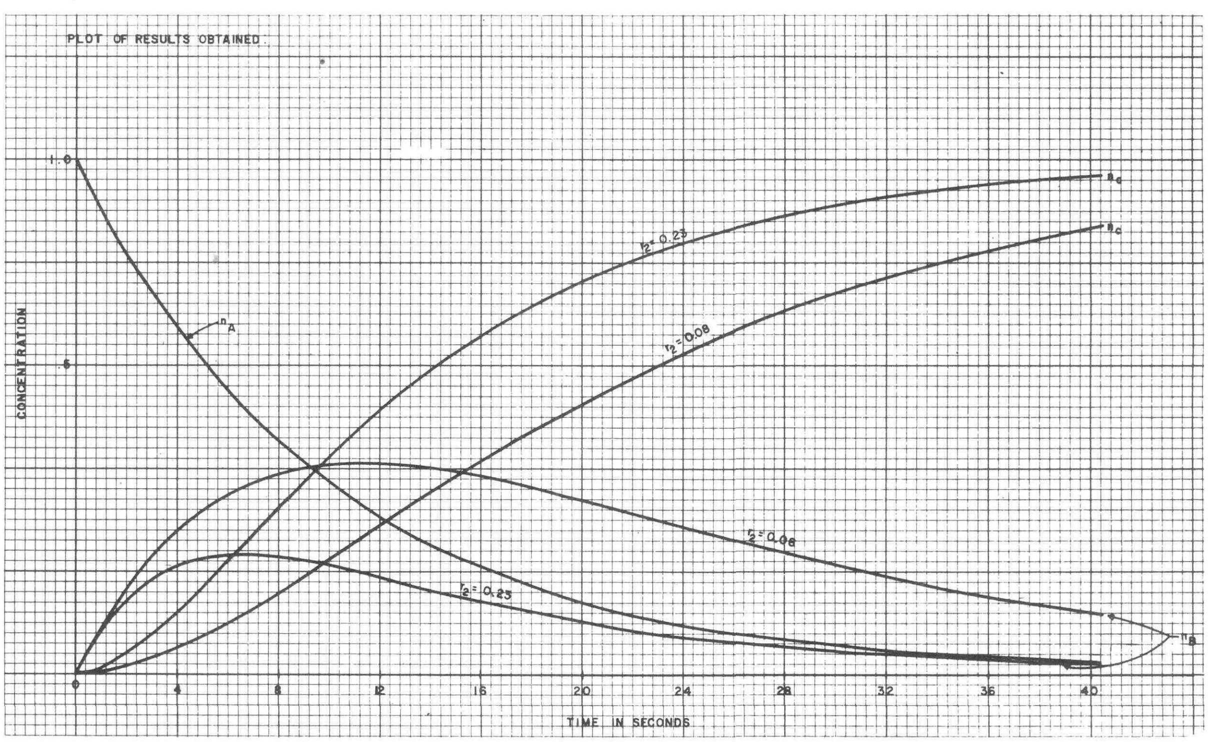
$$[10 \frac{dn_B}{dt}] = r_1 \; [10n_A] - r_2 \; [10n_B]$$

$$[10 \frac{dn_C}{dt}] = r_2 \; [10n_B]$$

Computer Diagram:



Plot of Results Obtained:

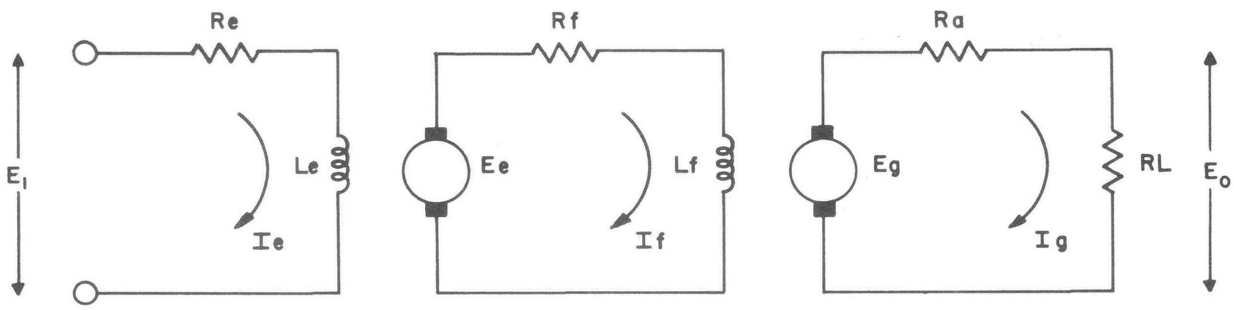


Plot of concentrations as a function of reaction time for different specific reaction rates.

DYNAMIC RESPONSE OF A SEPARATELY EXCITED GENERATOR

Problem Description:

To investigate the time response of a particular design of a separately excited generator for various changes in the exciter voltage with a constant resistive load.



Schematic Diagram of Separately Excited Generator

From the schematic the following equations can be written:

$$E_i = I_e R_e + L_e \frac{dI_e}{dt}$$

$$E_e = I_f R_f + L_f \frac{dI_f}{dt}$$

$$E_g = I_g (R_a + R_L)$$

$$E_o = \frac{R_L}{R_a + R_L} E_g$$

The armature voltage of the exciter E_e is proportional to the field current I_e, so that

$$E_e = K_e I_e$$

Similarly the emf of the generator E_g is proportional to the field current I_f.

$$E_g = K_g I_f$$

The equations to be programmed on the computer will then be :

$$\frac{dE_e}{dt} = E_i \frac{K_e}{L_e} - E_e \frac{R_e}{L_e}$$

$$\frac{dE_g}{dt} = E_e \frac{K_g}{L_f} - E_g \frac{R_f}{L_f}$$

$$E_o = \frac{R_L}{R_a + R_L} E_g$$

Maximum values :

$E_i = 200$ volts

$E_e = 200$ volts

$E_g = 500$ volts

$E_o = 500$ volts

Scaled Equations :

The variables are scaled as shown below to assure that the voltages representing their maximum values do not exceed ± 10 volts :

$$\left[\frac{1}{20}\frac{dE_e}{dt}\right] \qquad \left[\frac{1}{50}\frac{dE_g}{dt}\right]$$

$$\left[\frac{E_e}{20}\right] \qquad \left[\frac{E_g}{50}\right]$$

$$\left[\frac{E_i}{20}\right] \qquad \left[\frac{E_o}{50}\right]$$

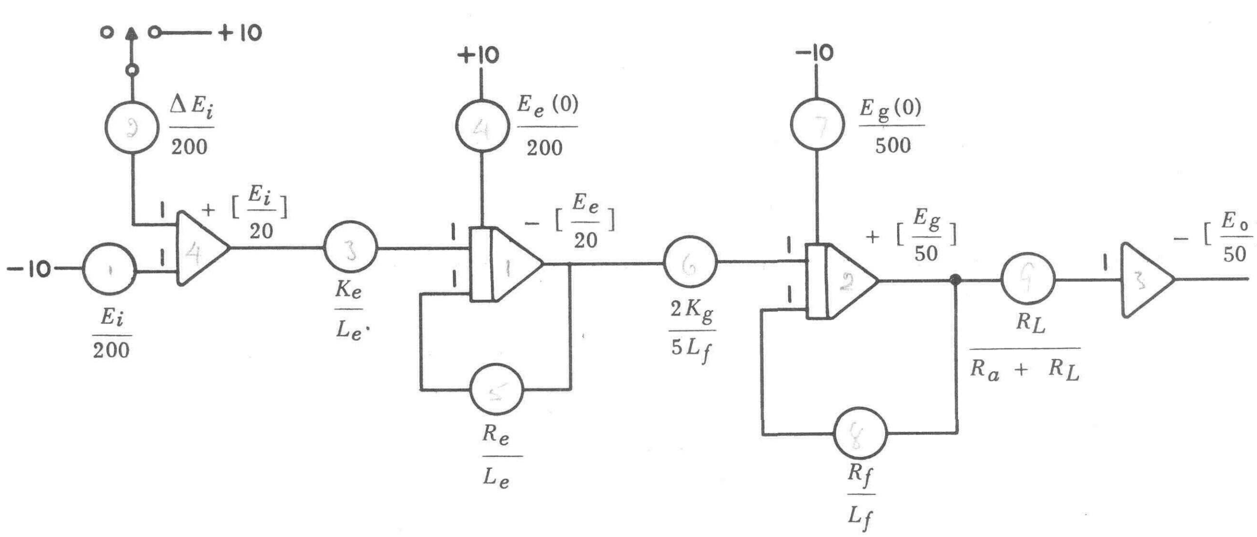
and the scaled equations are of the form :

$$\left[\frac{1}{20}\frac{dE_e}{dt}\right] = \left(\frac{K_e}{L_e}\right)\left[\frac{E_i}{20}\right] - \left(\frac{R_e}{L_e}\right)\left[\frac{E_e}{20}\right]$$

$$\left[\frac{1}{50}\frac{dE_g}{dt}\right] = \left(\frac{2}{5}\frac{K_g}{L_f}\right)\left[\frac{E_e}{20}\right] - \left(\frac{R_f}{L_f}\right)\left[\frac{E_g}{50}\right]$$

$$\left[\frac{E_o}{50}\right] = \left(\frac{R_L}{R_a + R_L}\right)\left[\frac{E_g}{50}\right]$$

Computer Diagram



Results of Computer Solution:

The response of the generator for changes in the exciter voltage is shown in Figure 1 and Figure 2. Note that the computer diagram is arranged such that the generator time response to changes in design parameters can be quickly determined.

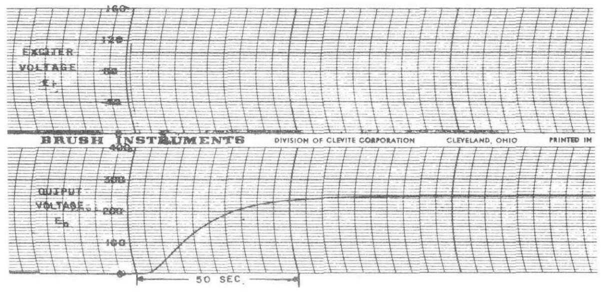


Figure 1: Response of Separately Excited Generator to Application of Exciter Voltage.

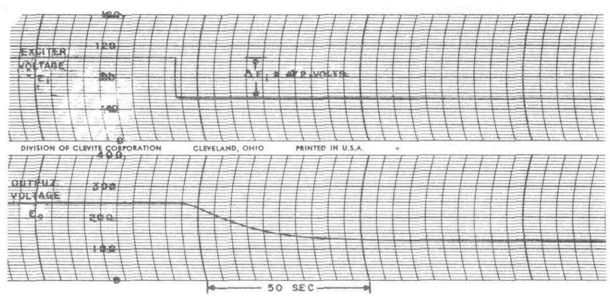


Figure 2: Response of Separately Excited Generator to Changes in Exciter Voltage.

PACE TR-10 ANALOG COMPUTER

APPLICATION NOTES

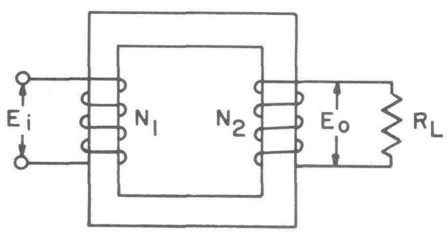
NUMBER **7**

TIME RESPONSE OF A TWO-WINDING TRANSFORMER

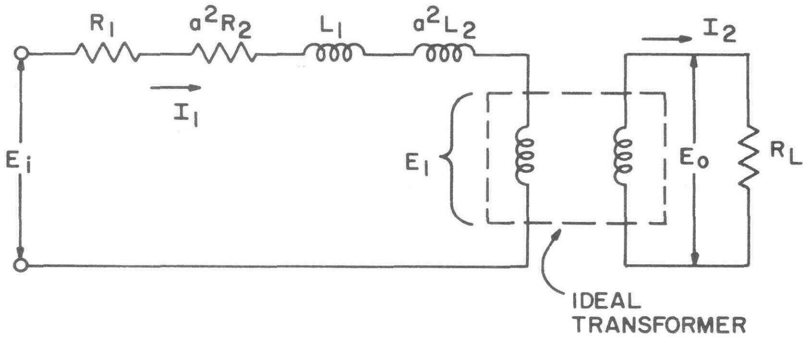
Problem Description:

To determine the time response of a particular design of a two-winding transformer for changes in the resistive load and input voltage.

Equations:



SCHEMATIC OF TWO-WINDING TRANSFORMER



SIMPLIFIED EQUIVALENT CIRCUIT OF TWO-WINDING TRANSFORMER

From the equivalent circuit the following primary and secondary voltage-equilibrium equations can be written:

$$E_i = (R_1 + a^2R_2)I_1 + (L_1 + a^2L_2)\frac{dI_1}{dt} + E_1$$

$$E_o = I_2R_L$$

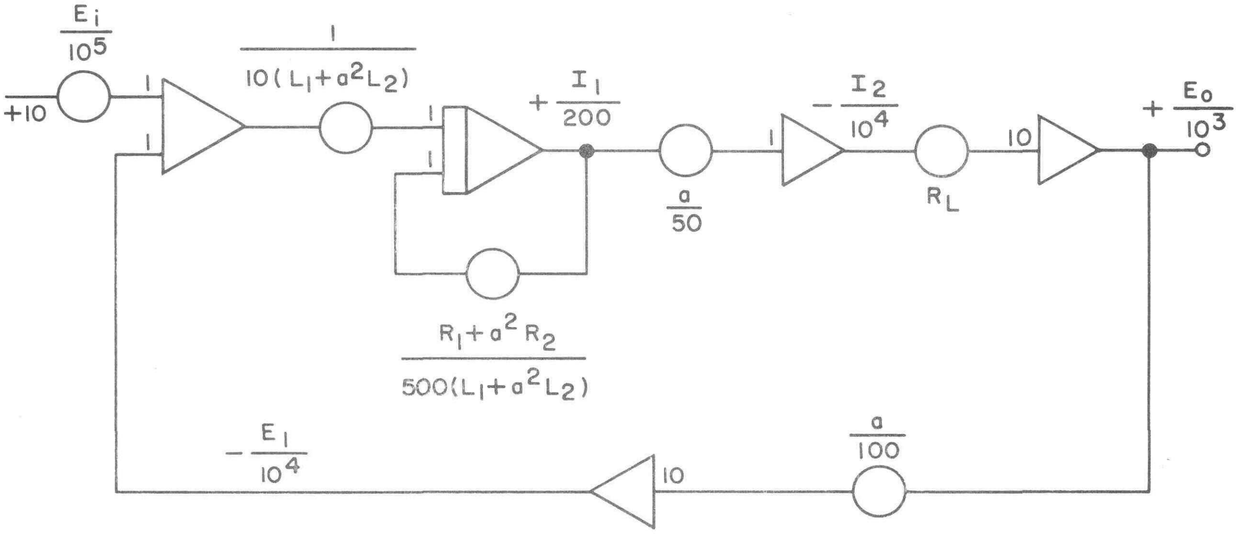
$$\frac{E_1}{E_o} = \frac{N_1}{N_2} = a$$

$$N_2I_2 = N_1I_1$$

where

R_1	=	Resistance of primary winding
R_2	=	Resistance of secondary winding
L_1	=	Inductance of primary winding
L_2	=	Inductance of secondary winding
a	=	Transformer turns ratio
E_i, E_o, E_1	=	RMS voltages
I_1, I_2	=	RMS currents

Computer Diagram:



Results of Computer Solution:

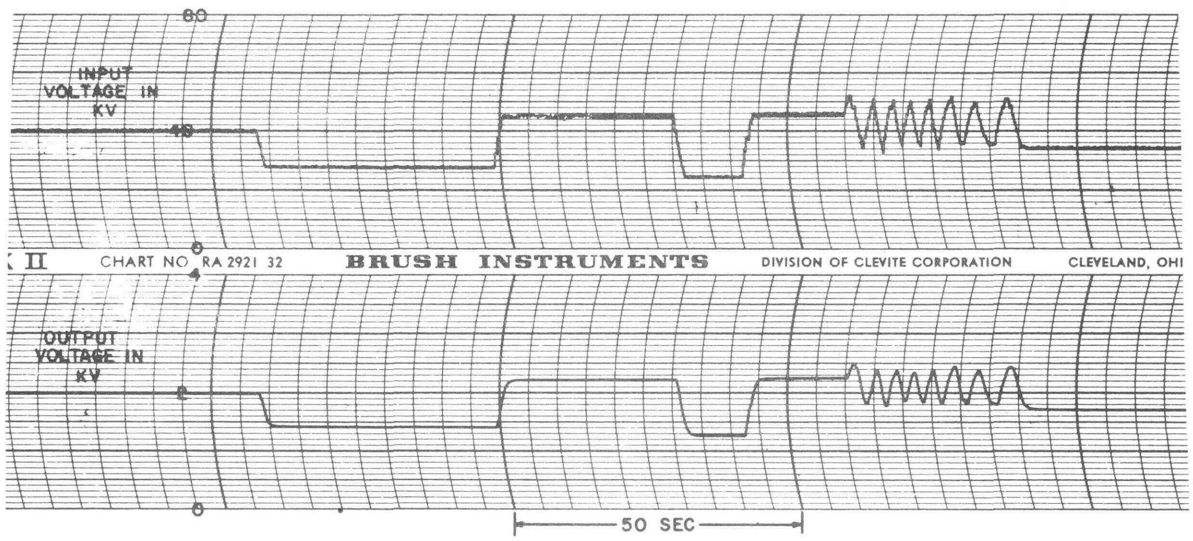


Figure 1: Response of Output Voltage of Two-winding Transformer for Changes in Input Voltage.

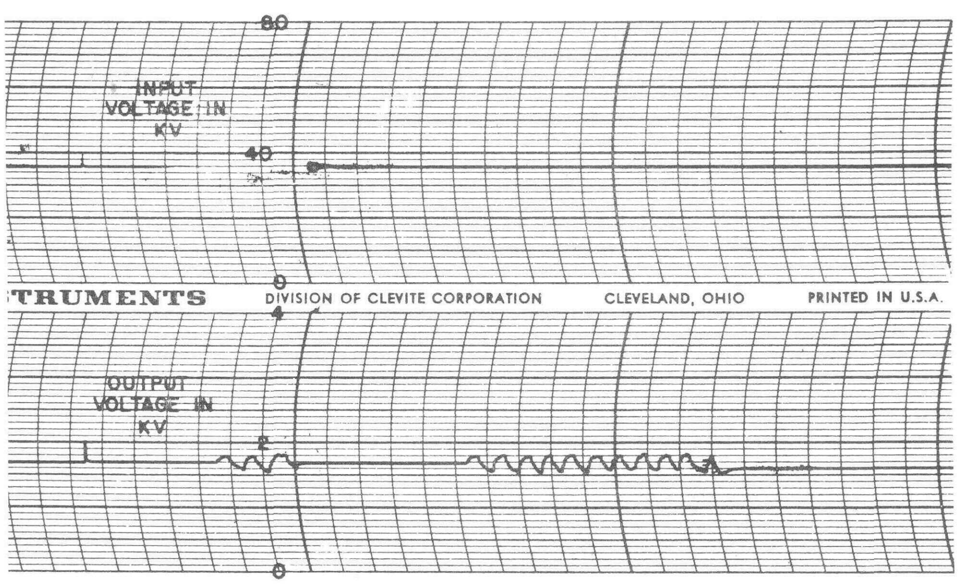


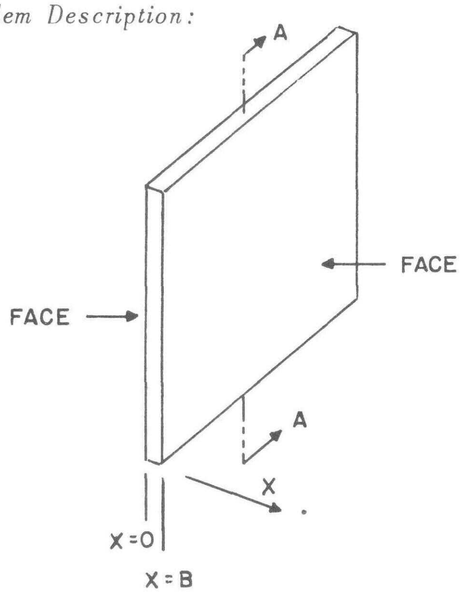
Figure 2: Response of Output Voltage of Two-winding Transformer for Changes in Resistive Load.

INVESTIGATION OF UNSTEADY-STATE HEAT CONDUCTION

Discussion:

Many engineering problems involve the sudden heating or quenching of large slabs of material. In the annealing or heat treating of metals, plastics, glass, and rubber, for example, an accurate knowledge of the temperature vs. time and thickness relationships is required. Consider the following representative problem.

Problem Description:



Determine the temperature distribution along the X direction of a long wide slab. The slab is initially at a high temperature (T1) and is suddenly quenched in a large ice-water bath at 0°C. Assume that the temperature in the slab varies in the X direction only.

Equations:

With the assumption made above, the unsteady-state heat transfer can be expressed by the following equation:

$$\frac{\partial T}{\partial t} = \frac{k}{C_p \rho} \frac{\partial^2 T}{\partial x^2}$$

The boundary conditions are, for $T = T(x, t)$

$T(x, 0) = T'$		$0 < x < B$
$T(0, t) = 0$		$t > 0$
$T(B, t) = 0$		$x = B$

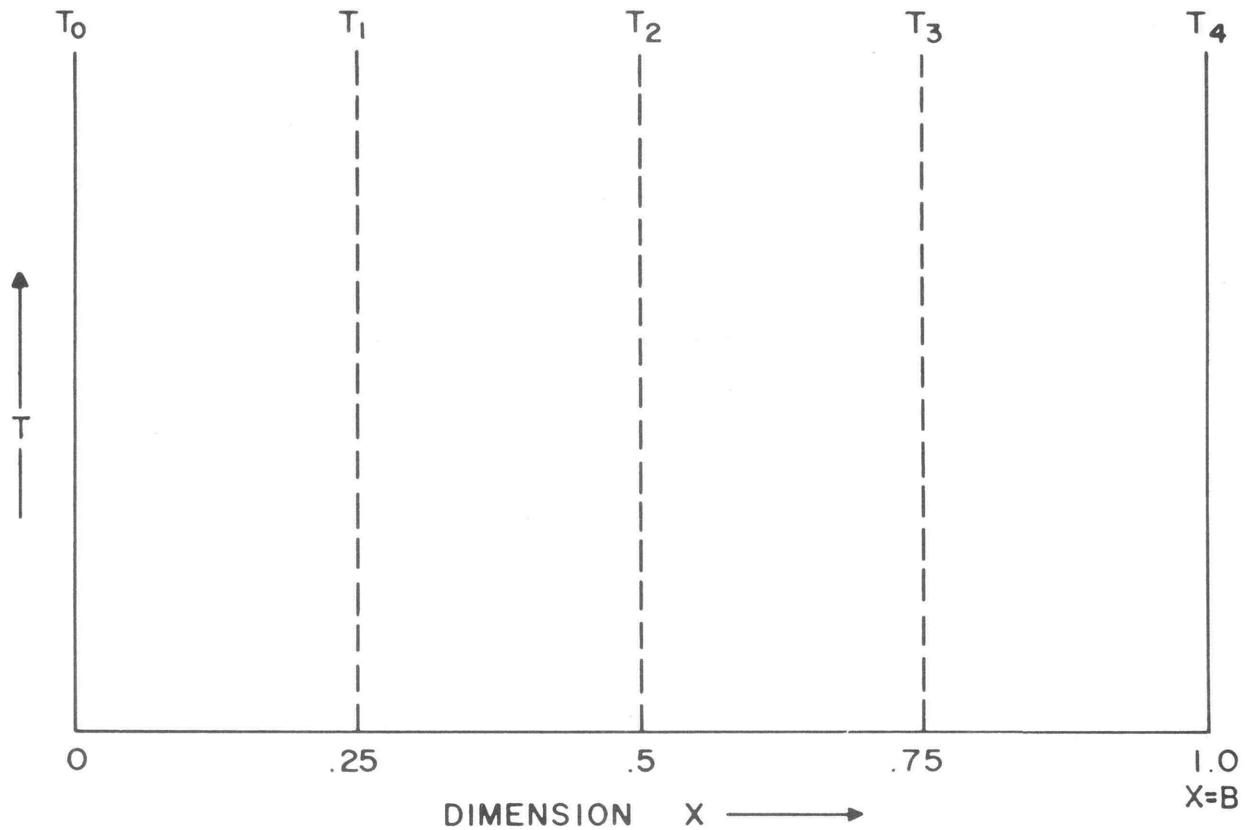
Where

- T = temperature in degrees C.
- t = time
- k = thermal conductivity of material
- Cp = specific heat of material (ave)
- ρ = density of material (ave)

Using a finite-difference approximation of the second derivative of temperature with respect to the space dimension x, the second central difference becomes ....

$$\frac{\partial^2 T_\eta}{\partial x^2} = \frac{1}{\Delta X^2}\left(T_{\eta+1} - 2T_\eta + T_{\eta-1}\right)$$

where the subscripts $\eta + 1$, η, $\eta - 1$ are referred to the number of cross section of the slab.



The use of the finite-difference approach is due to the fact that the Analog Computer can continuously integrate with respect to only a single independent variable at any one time. This approach is equivalent to dividing the slab into sections as shown by the sketch and solving for the temperature at each section.

Using the finite-difference approach the Analog Computer must simultaneously solve the following ordinary differential equations

$$\left[\frac{1}{10}\frac{dT_0}{dt}\right] = \left(\frac{k}{C_p \rho \Delta x^2}\right)\left\{\left[\frac{T_1}{10}\right] - (2)\left[\frac{T_0}{10}\right]\right\} \qquad T_{-1}(t) = 0$$

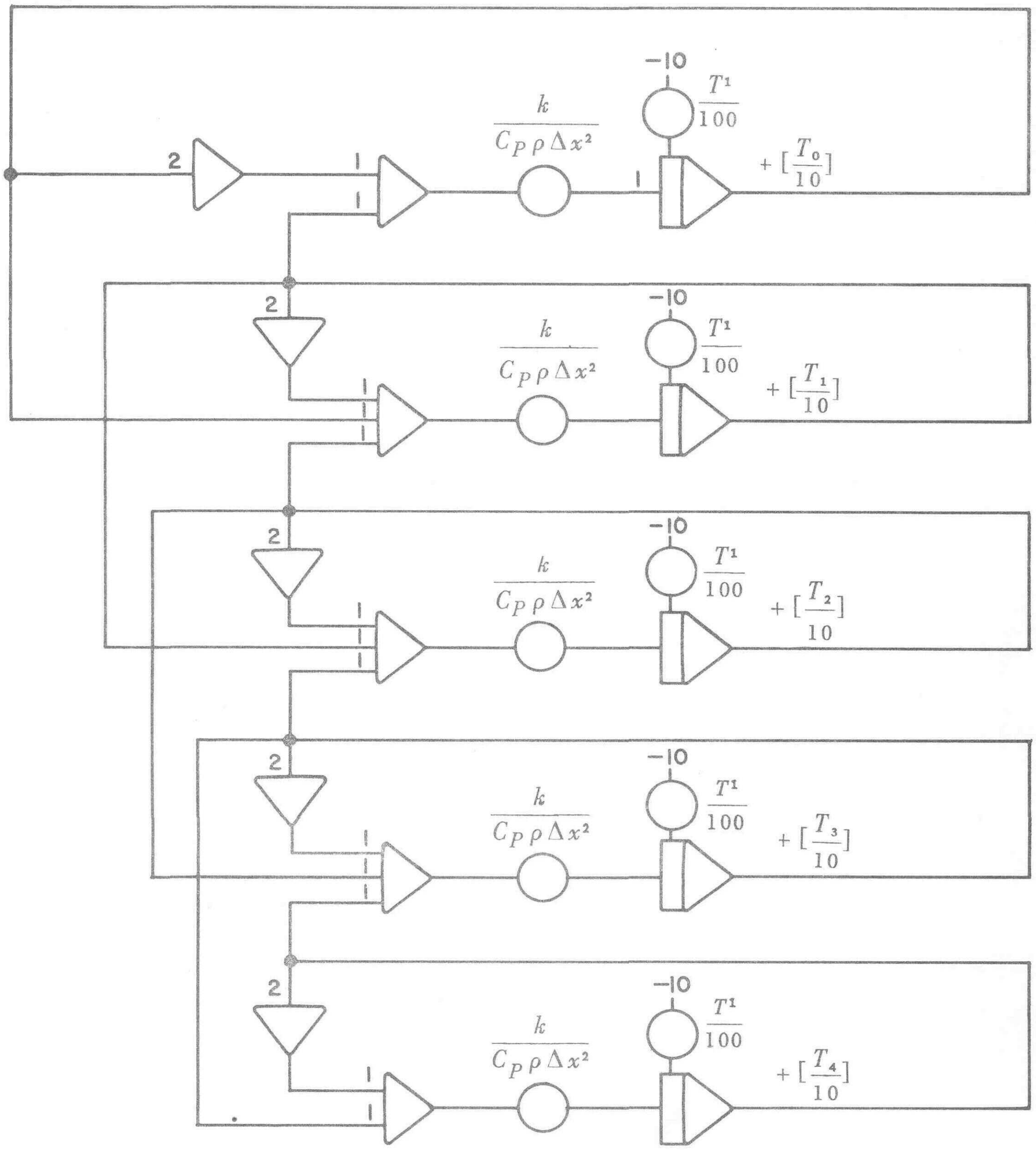
$$\left[\frac{1}{10}\frac{dT_1}{dt}\right] = \left(\frac{k}{C_p \rho \Delta x^2}\right)\left\{\left[\frac{T_2}{10}\right] - (2)\left[\frac{T_1}{10}\right] + \left[\frac{T_0}{10}\right]\right\}$$

$$\left[\frac{1}{10}\frac{dT_2}{dt}\right] = \left(\frac{k}{C_p \rho \Delta x^2}\right)\left\{\left[\frac{T_3}{10}\right] - (2)\left[\frac{T_2}{10}\right] + \left[\frac{T_1}{10}\right]\right\}$$

$$\left[\frac{1}{10}\frac{dT_3}{dt}\right] = \left(\frac{k}{C_p \rho \Delta x^2}\right)\left\{\left[\frac{T_4}{10}\right] - (2)\left[\frac{T_3}{10}\right] + \left[\frac{T_2}{10}\right]\right\}$$

$$\left[\frac{1}{10}\frac{dT_4}{dt}\right] = \left(\frac{k}{C_p \rho \Delta x^2}\right)\left\{- (2)\left[\frac{T_4}{10}\right] + \left[\frac{T_3}{10}\right]\right\} \qquad T_5(t) = 0$$

Computer Diagram:



Results of Computer Solution:

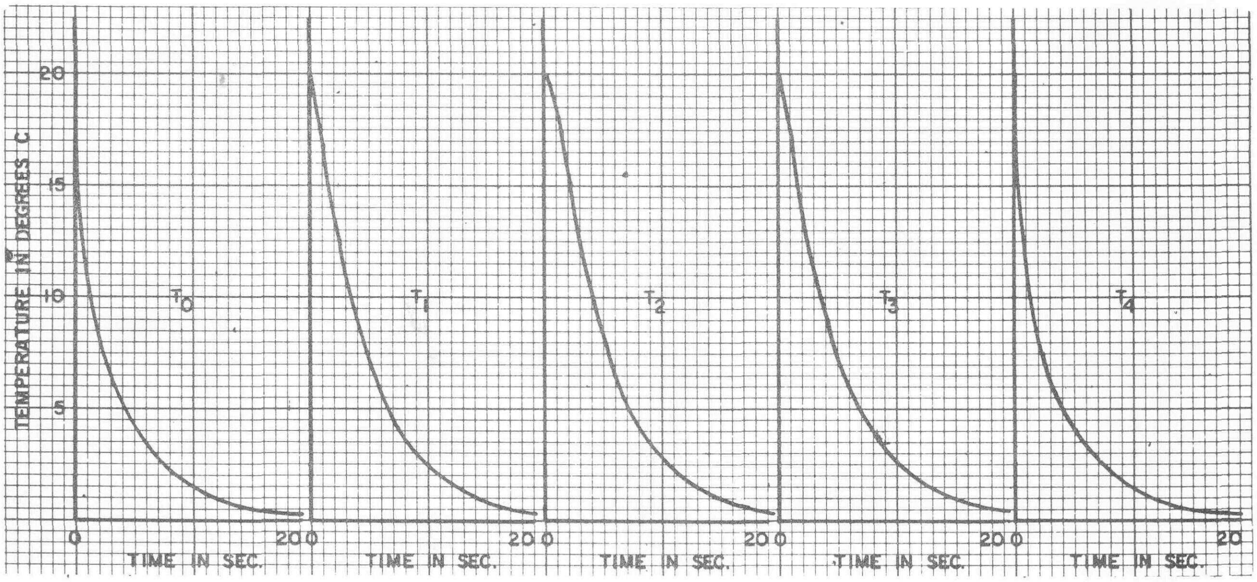


Figure 1: Temperature Change vs. Time At Several Points Along X Direction.

PACE
TR-10
ANALOG COMPUTER

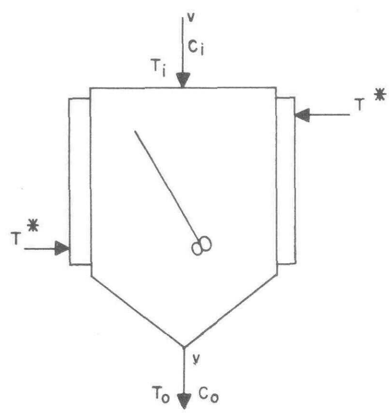
APPLICATION NOTES
NUMBER 10

INVESTIGATION OF THE PROCESS PARAMETERS AFFECTING THE CONTROL OF A STIRRED TANK REACTOR

Discussion:

The continuous stirred tank reactor is an essential part of many chemical processes. The problem below shows, in simplified form, how a model of a CSTR may be set up and studied.

Problem Description:



To investigate the effect of parameter changes on the time response of the output concentration of a Stirred Tank Reactor.

Equations:

The reaction taking place is

$$A \overset{k}{\rightarrow} B$$

When the specific reaction rate is defined as

$$k = k_{gm} + b\,T_0$$

Material Balance:

Material in	$v\ C_i$
Material out	$v\ C_0$
Material reacted	$k\ V_T C_0$
Material accumulated	$V_T \frac{dC_0}{dt}$

Summing:

$$\frac{dC_0}{dt} = \frac{v}{V_T} C_i - \frac{v}{V_T} C_0 - k\,C_0$$

Heat Balance:

Heat in	$v \rho C_P T_i$
Heat out	$v \rho C_P T_0$
Heat transferred	$h A^*(T_0 - T^*)$
Heat due to reaction	$-Q\, V_T\, k\, C_0$
Heat accumulated	$\rho V_T C_P \frac{dT_0}{dt}$

EAI ELECTRONIC ASSOCIATES INC. *Long Branch, New Jersey*

PRINTED IN U.S.A. AF941 BULLETIN #AN 930-1

Summing:

$$\frac{dT_o}{dt} = \frac{v}{V_T} T_i - \frac{h A}{\rho V_T C_P} (T_o - T^*) - \frac{Q k C_o}{\rho C_P} - \frac{v}{V_T} T_o$$

where

T_o, T_i, T^*	=	temperature
v	=	flow rate of material
V_T	=	reactor volume
t	=	time
h	=	heat transfer coefficient
A^*	=	effective area of heat transfer
ρ	=	average density of material
Q	=	heat due to reaction
C_P	=	average specific heat
C_i	=	input concentration
C_o	=	output concentration

Scaled Equations:

The variables are scaled as shown below so that the computer problem voltages are as high as possible but do not exceed ± 10 volts.

$[10C_i]$ $[10K]$

$[10C_o]$

$[\frac{T_i}{10}]$

$[\frac{T_o}{10}]$

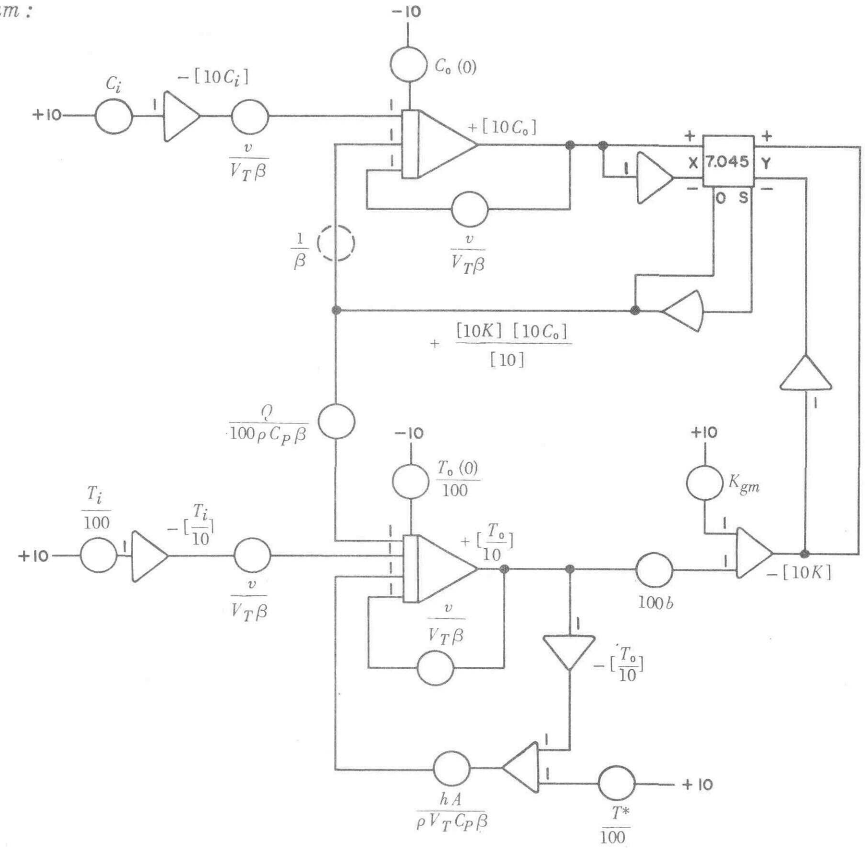
$[\frac{T^*}{10}]$

$$[10K] = 10\,K_{gm} + (100b)\,[\frac{T_o}{10}]$$

$$[10\frac{dC_o}{dt}] = \left(\frac{v}{V_T}\right)[10C_i] - \left(\frac{v}{V_T}\right)[10C_o] - \frac{[10K]\,[10C_o]}{[10]}$$

$$[\frac{1}{10}\frac{dT_o}{dt}] = \left(\frac{v}{V_T}\right)[\frac{T_i}{10}] - \left(\frac{h A}{\rho V_T C_P}\right)\left([\frac{T_o}{10}] - [\frac{T^*}{10}]\right) - \left(\frac{Q}{100\rho C_P}\right)\frac{[10K]\,[10C_o]}{[10]} - \left(\frac{v}{V_T}\right)[\frac{T_o}{10}]$$

Computer Diagram:



Note that the inputs to the integrators have been multiplied by a time scale factor $\frac{1}{\beta}$. For $\beta < 1$ the computer solution will be β times faster than the response of the real physical system.

Results of Computer Solution:

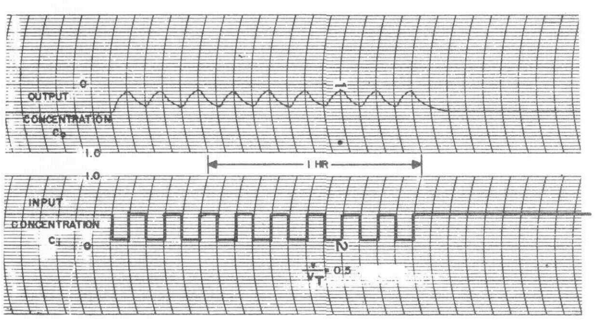


Figure 1a: Response of Output Concentration for Disturbance to Input Concentration. ($v/V_T = 0.5$)

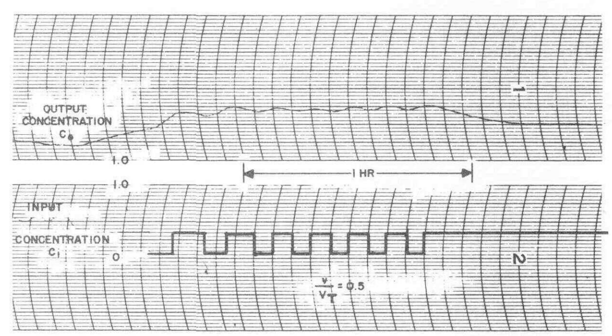
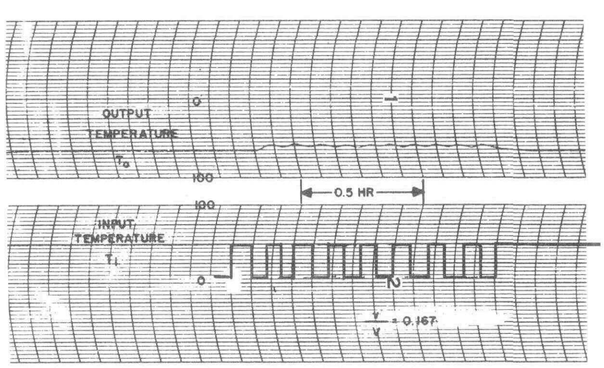


Figure 1b: Response of Output Concentration for Disturbance to Input Concentration ($v/V_T = 0.167$)



$v/V_T = 0.5$

$T_o(0) = 65.0$

$C_i = 0.45$

$T_i = 50$

$T^* = 0$

$C_o(\theta) = 0.4$

Figure 1c: Response of Output Temperature of Reactor for Disturbance to Temperature of Material Entering Reactor. ($v/V_T = 0.167$)

PACE TR-10 ANALOG COMPUTER

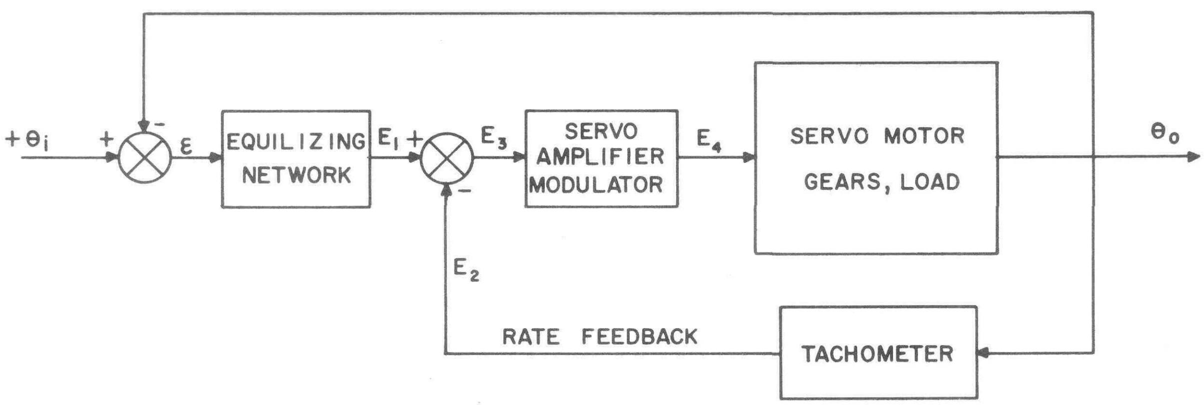
APPLICATION NOTES

NUMBER 11

INVESTIGATION OF THE OUTPUT RESPONSE OF A POSITIONAL SERVO SYSTEM

Problem Description:

To determine the output response of a positional Servo System to input disturbances for changes in the system parameters.



FUNCTIONAL BLOCK DIAGRAM OF A SERVOMECHANISM

Equations:

Error Device $\theta_i - \theta_o = \epsilon$

Equalizing Network $\frac{E_1}{\epsilon} = K_1 \frac{Tp + 1}{aTp + 1}$

Servo Amplifier $E_4 = K_2 E_3 = K_2 (E_1 - E_2)$

Servo Motor, Gears, Load

$$\frac{\theta_o}{E_4} = \frac{K_3 N}{(I_L + N^2 I_m) p^2 + N^2 R + K}$$

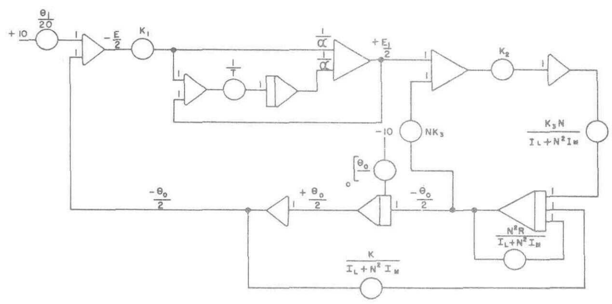
Tachometer $\frac{E_2}{\theta_o} = N k_3 p$

EAI ELECTRONIC ASSOCIATES INC. Long Branch, New Jersey

PRINTED IN U.S.A. AF941

BULLETIN # AN 931-1

Computer Diagram:



Results of Computer Solution:

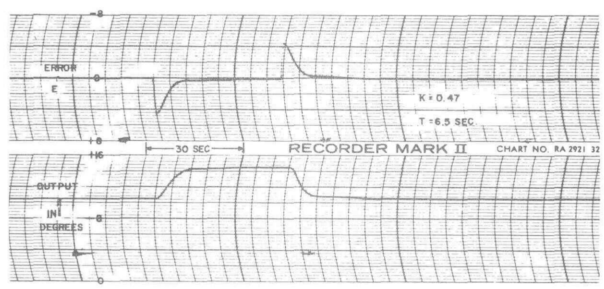


Figure 1a: Response of Servo for step input (k_1 = 0.47, T = 6.5 sec.)

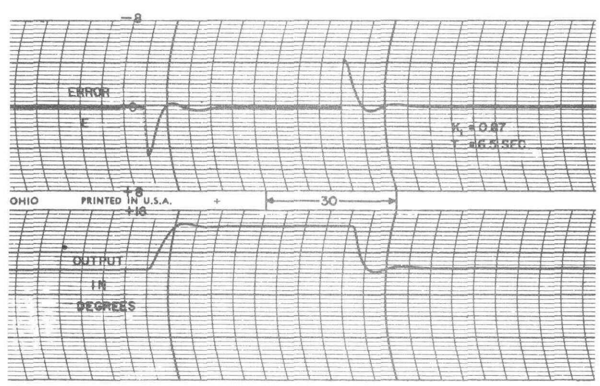


Figure 1b: Response of Servo for step input (k_1 = 0.87, T = 6.5 sec.)

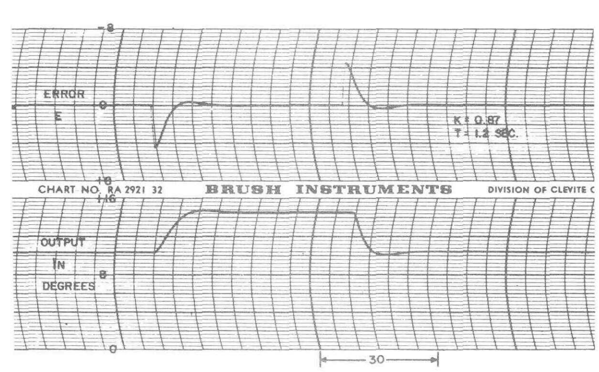


Figure 1c: Response of Servo for step input (k_1 = 0.87, T = 1.2 sec.)

INVESTIGATION OF THE RESPONSE OF A NUCLEAR REACTOR

Problem Description:

To investigate the time response of a Design of a Nuclear Reactor for changes in the Reactor multiplication factor δk.

Equations:

The equations[1] representing the performance of a Nuclear Reactor are as follows:

$$\frac{dn}{d\theta} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{i=6} \lambda_i C_i$$

$$\frac{dC_i}{d\theta} = \frac{\beta_i}{l^*} n - \lambda_i C_i$$

To represent a Nuclear Reactor with five delay groups the following differential equations must be solved simultaneously:

$$\frac{dn}{d\theta} = \frac{\delta k - \beta}{l^*} n + \sum_{i=1}^{i=5} \lambda_i C_i$$

$$\frac{dC_1}{d\theta} = \frac{\beta_1}{l^*} n - \lambda_1 C_1$$

$$\frac{dC_2}{d\theta} = \frac{\beta_2}{l^*} n - \lambda_2 C_2$$

$$\frac{dC_3}{d\theta} = \frac{\beta_3}{l^*} n - \lambda_3 C_3$$

$$\frac{dC_4}{d\theta} = \frac{\beta_4}{l^*} n - \lambda_4 C_4$$

$$\frac{dC_5}{d\theta} = \frac{\beta_5}{l^*} n - \lambda_5 C_5$$

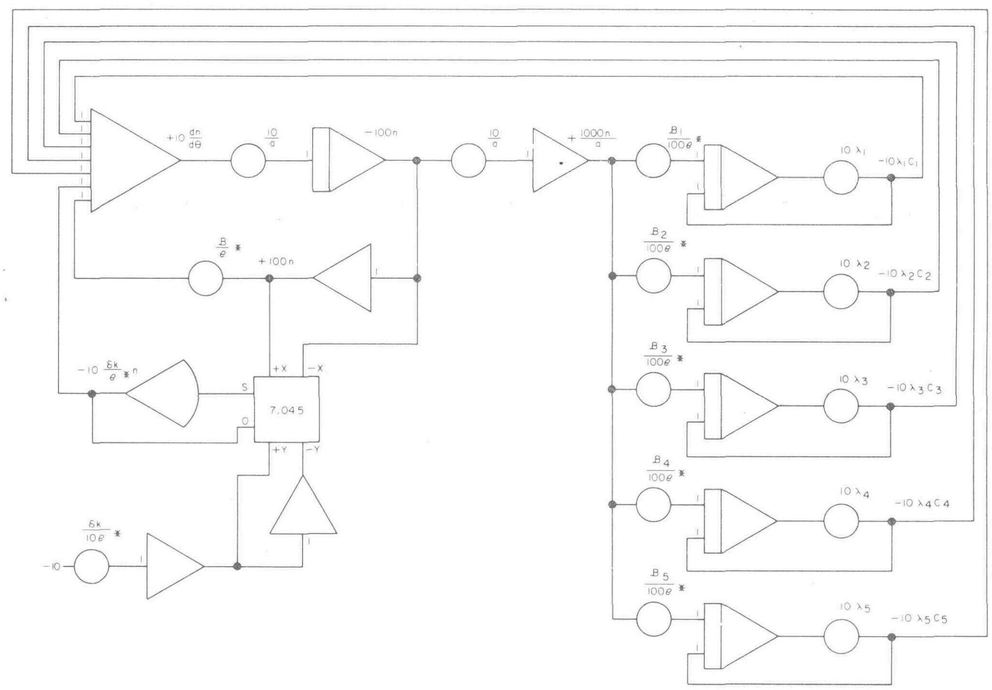
1. M.A. Schultz, "*Control of Nuclear Reactors and Power Plants*", McGraw Hill Book Co., Inc., New York, 1955.

EAI ELECTRONIC ASSOCIATES INC. Long Branch, New Jersey

PRINTED IN U.S.A. AF941

BULLETIN #AN 932

Computer Diagram:



Results of Computer Solution:

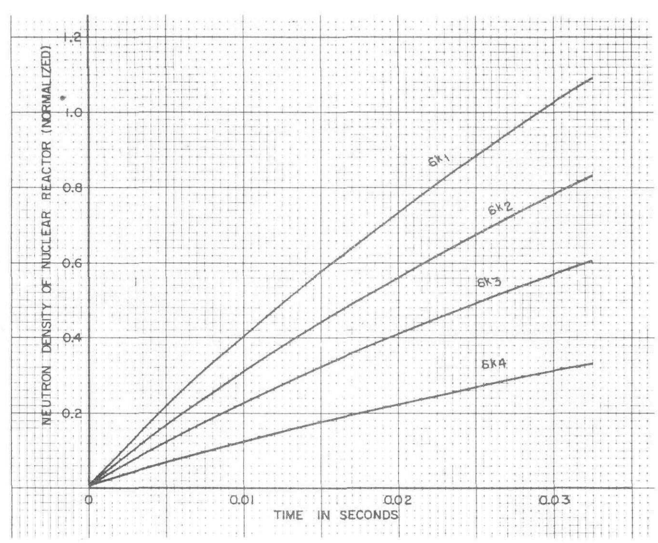


Figure 1: Response of Nuclear Reactor for Changes in δk.

PACE TR-10 ANALOG COMPUTER

APPLICATION NOTES

NUMBER 9

INVESTIGATION OF THE REACTION OF SODIUM VAPOR DIFFUSING INTO A HALIDE

Discussion:

Many chemical reactions are so rapid that diffusion of the reactants and products around the reacting zone will completely control the system behavior. Such is the case when sodium vapor reacts with a halide. In the problem below, vaporized sodium was admitted through a nozzle into a large reactor filled with a halide gas. The equations describing this process demand a trial and error type method of solution.

Problem Description:

To investigate the effect of changes in design parameters on the reaction of Sodium Vapor diffusing into a Halide.

Equations:

The following equation[1] describes the reaction of Sodium Vapor diffusing into a Halide:

$$\frac{d^2V}{dx^2} = \frac{KV(V + x + x_1 - 1)}{x + x_1}$$

The boundary conditions are

$$V = 1 - x_1 \text{ at } x = 0$$

$$V \to 0 \qquad \text{as } x \to \infty$$

The parameters $(1 - x_1)$ and K correspond to a measure of the nozzle opening used in a reactor and chemical rate of reaction, respectively. The solution to the equation is obtained when dV/dx and V become zero simultaneously.

Scaled Equation:

The independent variable x will be replaced by time so that $x = t$. The variables are scaled as follows:

$$10\frac{d^2V}{dt^2} \qquad 0 \leqslant t \leqslant +10$$

$$10\frac{dV}{dt} \qquad 0 \leqslant x_1 \leqslant +10$$

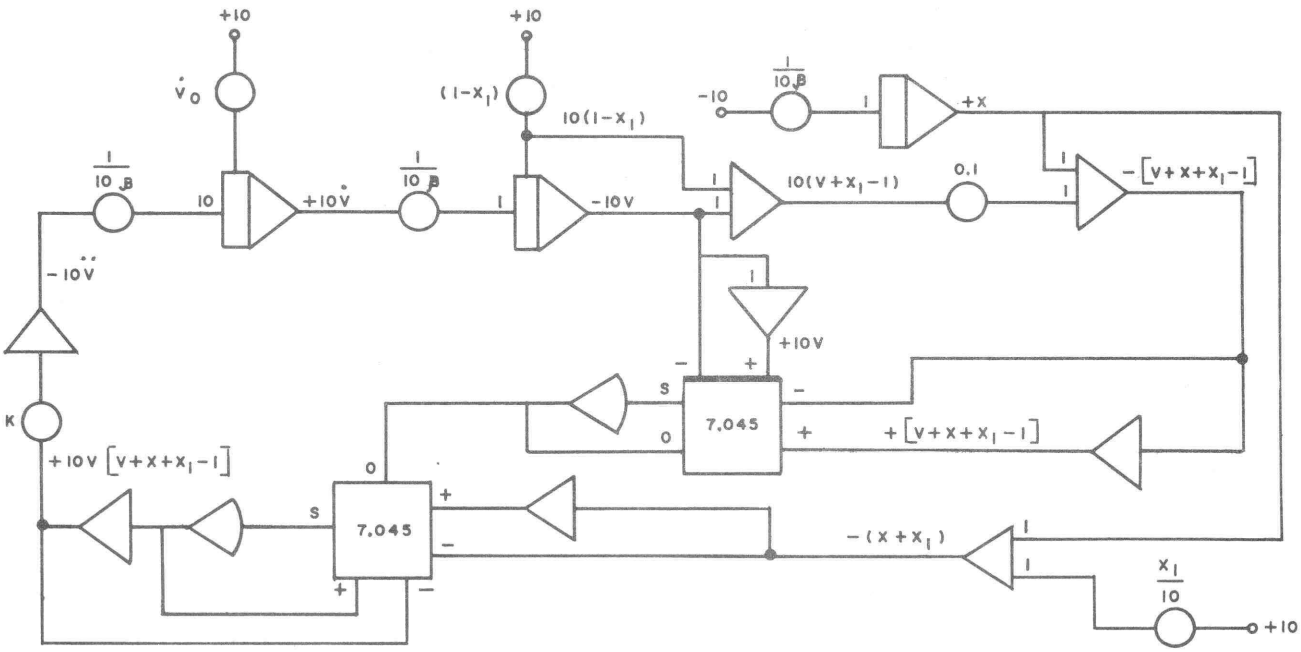
$$[10V]$$

The scaled voltages do not exceed the maximum permissable computer voltages, ± 10 volts, and permit the following scaled equation to be written: -

1. *"Mathematic Tables and Other Aids to Computation"*, Physics Abstracts, Vol. IX. July, 1955, pp. 112 - 116

$$\left[10\,\frac{d^2V}{dt^2}\right] = \frac{\left\{\left(\frac{1}{10}\right)[10V] + [t] + [x_1] - 1\right\}\frac{[10V]}{[10]}}{[t] + [x_1]}\,[10]\,(K)$$

Computer Diagram:



Note that the inputs to the integrators have been multiplied by a time scale factor $\frac{1}{\beta}$. For $\beta > 1$ the computer solution will be β times slower than the real reaction of the sodium vapor.

Results of Computer Solution:

The results of several trial runs are shown in Figure 1. From these trial solutions it can be seen that if $\dot{V}_0$ is too large, V will change sign before $\dot{V}$. If $\dot{V}_0$ is too small $\dot{V}$ changes sign before V. The objective in choosing a $\dot{V}_0$ is to cause both V and $\dot{V}$ to go to zero simultaneously. The best technique to use is to find a "small" value of $\dot{V}_0$ and then a "large" value of $\dot{V}_0$, and approach the "small" value by successively reducing $\dot{V}_0$ until the correct solution is obtained.

Typical final solutions for V as a function of the independent variable are shown in Figure 2.

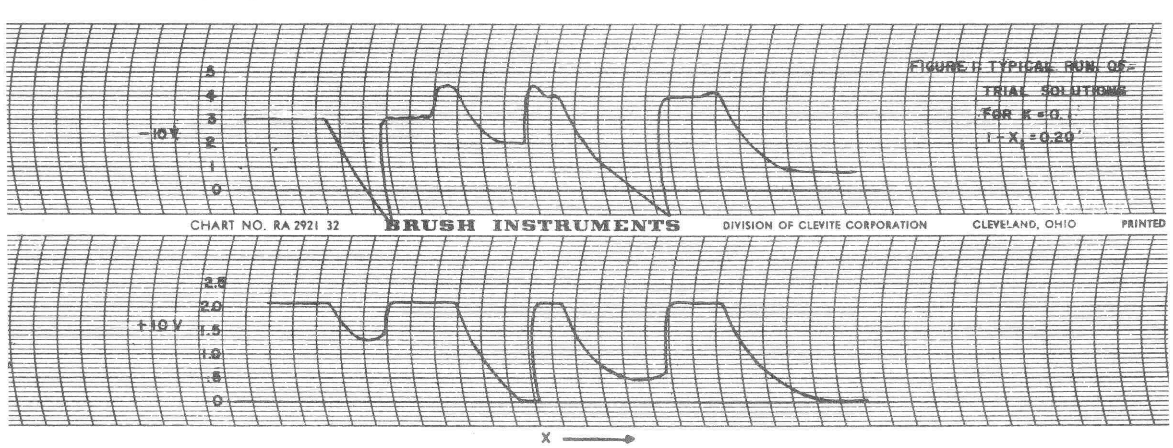


Figure 1: Typical run of trial solution for K = 0.1, 1 – x_1 = 0.20.

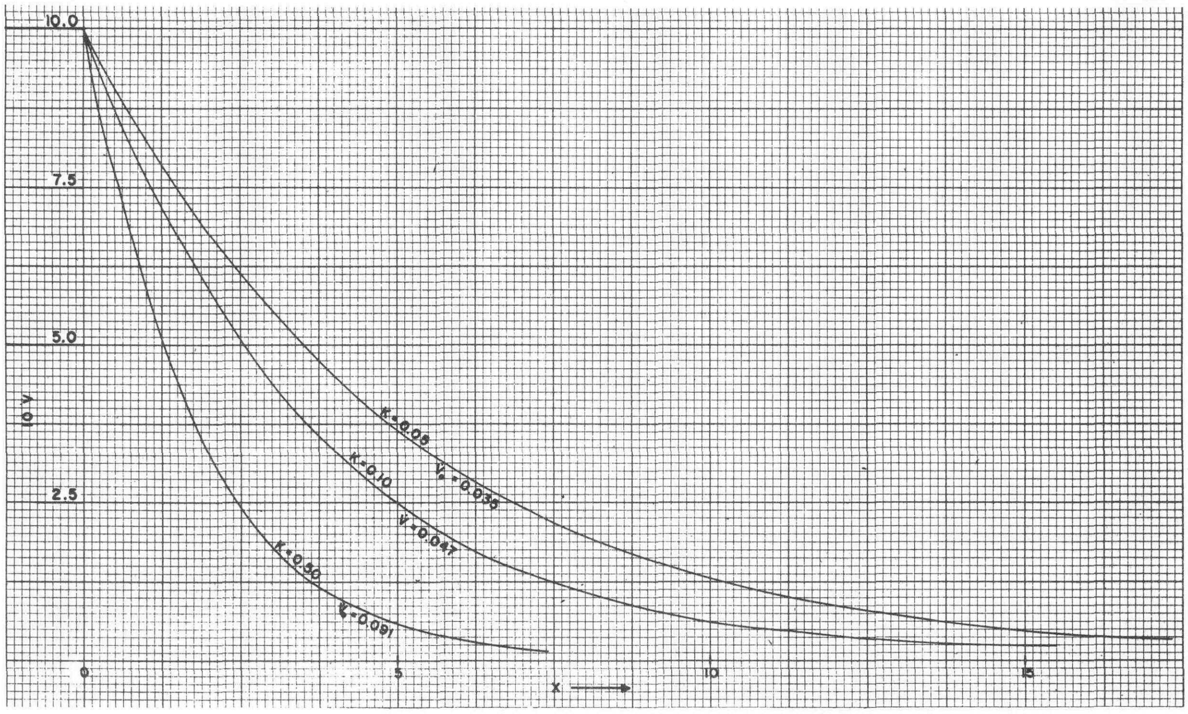


Figure 2: Typical Results of Calculations of the Reaction of Sodium Vapor diffusing into a Halide.